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On local fluctuations of stable moving average processes

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Let $X(t)=\int_{-\infty}^{t} f(t-s)\,\mathrm{d}Z(s)$ be a symmetric stable moving average process of index α, $1<\alpha\leqslant 2$. It is proved that when f has a jump discontinuity at a point or when $f(x)\to 0$ slowly as $x\downarrow 0$, then almost every sample function of $X(t)$, $t\in\mathbb{R}$, is a Janik (J_1) function with infinite γ-variation, $\gamma\in[1,\alpha)$.

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stable processes * moving average processes * local time * Jarnik functions * Hölder condition * γ-variations

1. Introduction

Let $X(t)$, $t\in\mathbb{R}$, $\mathbb{R}$ is the set of real numbers, be a measurable nonanticipating moving average stable stochastic process of index α, $1<\alpha\leqslant 2$, over a probability space $(\Omega, \mathcal{F}, P)$ taking values in $\mathbb{R}$, namely

$$X(t)=\int_{-\infty}^{t} f(t-s)\,\mathrm{d}Z(s), \quad t\in\mathbb{R}, \tag{1.1}$$

where $f:\mathbb{R}\to\mathbb{R}$ is a deterministic function which is zero on $(-\infty,0)$, and $\int_0^\infty |f(t)|^\alpha\,\mathrm{d}t<\infty$. In (1.1), $Z(t)$, $t\in\mathbb{R}$, is the real α-stable noise generated by Lebesgue measure over $(\Omega, \mathcal{F}, P)$, and the stochastic integral is defined in the weak sense, Cambanis and Soltani (1984). There are extensive studies on the behaviour of sample functions of stable Processes, see Rosinski (1986), Nolan (1989) for the references. In order to study the sample functions of moving average processes, different authors have looked on the local behaviour of the function f. Rosinski (1986, Theorem 5.1) proved that continuity of f is necessary for the almost sure continuity of sample functions of the process $X(t)$. Our main object in this article is to study the local fluctuations of sample functions of the process $X(t)$. We will prove that when the function f has a jump discontinuity or when $f(x)\to 0$ slowly as $x\downarrow 0$ a continuous

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version of local time exists. γ-variations of the sample functions are infinite and Hölder condition of certain order is satisfied at no points, details are in Theorem 3.4 and Theorem 4.2. The work is organised as follows; next section contains preliminaries. The main theorem is in Section 3, and Hölder conditions and γ-variations of the sample functions are discussed in Section 4.

2. Preliminaries and notations

Let $F(t)$, $0 \leq t \leq 1$, be a real Borel function and let $I = [a, b]$ be a closed interval in $[0, 1]$. For $t \in I$, Put

$$\mu_I(t, B) = \lambda(F^{-1}(B) \cap [a, t]), \quad B \in \mathscr{B}(R),$$

where λ is the Lebesgue measure on the real Borel σ-field $\mathscr{B}([0, 1])$. The measure $\mu_I(t, \cdot)$ is called the occupation measure and $\mu_I(t, B)$ may be interpreted as the amount of time spent by F in B during the time period $[a, t]$. If $\mu_I(b, \cdot)$ is absolutely continuous with respect to m, the Lebesgue measure on $\mathscr{B}(\mathbb{R})$, we call $\alpha_I(t, x) \equiv \mathrm{d}\mu_I(t, x)/\mathrm{d}m(x)$, $t \in I$, $-\infty < x < \infty$, the local time of F on I. For $I = [0, 1]$, the occupation measure and the local time on I are denoted by $\mu(t, \cdot)$, $\alpha(t, x)$ respectively. The notion of local time was first introduced by Paul Levy for studying the Brownian trajectories. The theory of local time is well developed. A review of results until 1980 is contained in the work of Geman and Horowitz (1980). In Section 3 we will make use of Geman's results in Geman (1976). Indeed if the function F has a local time $\alpha(t, x)$ on $[0, 1]$ which is continuous in t for a.e. x, then by theorem A of Geman,

(a) $\quad \operatorname{ap-lim}_{s \to t} |(F(s) - F(t))/(s - t)| = +\infty \quad$ for a.e. $t \in [0, 1]$,

(b) $\quad L_t = \{s \in [0, 1] \mid F(s) = F(t)\}$ is uncountable for a.e. $t \in [0, 1]$.

For the definition of approximate limit (ap-lim) see Geman and Horowitz (1980, p. 22). If (a) is satisfied then F is called a Jarnik function (J_1), and for a.e. $t \in [0, 1]$, F fails to have finite derivatives or approximate derivatives Geman and Horowitz (1980, p. 15).

If $X(t, \omega)$, $t \in [0, 1]$, is a stochastic process, then for each $\omega \in \Omega$, the trajectory $t \to X(t, \omega)$ is a Borel function and the occupation measure $\mu_I(t, \cdot)$ and the occupation density $\alpha_I(t, x)$ (if it exists) depend on ω. In Geman (1976) it is also proved that if for a stochastic process $X(t, \omega)$, $t \in [0, 1]$, $\omega \in \Omega$,

$$\int_0^1 \sup_{\varepsilon > 0} \frac{1}{\varepsilon} P(|X(s) - X(t)| \leq \varepsilon)\, \mathrm{d}s < \infty \quad \text{for a.e. } t \in [0, 1], \tag{2.1}$$

then with probability one, $X(t, \omega)$ has a local time which is continuous in t for a.e. x. In the Gaussian case (2.1) is equivalent to

$$\int_0^1 \frac{\mathrm{d}s}{[E(X(t) - X(s))^2]^{1/2}} < \infty \quad \text{for a.e. } t \in [0, 1], \tag{2.2}$$

which is implied by

$$\int_0^1 \int_0^1 \frac{\mathrm{d}s\,\mathrm{d}t}{[E(X(t)-X(s))^2]^{1/2}} < \infty. \tag{2.3}$$

The latter is the Berman's sufficient condition for the existence of a square integrable (w.r.t. dx) local time, Berman (1969a, p. 283).

When $X(t, \omega)$ is a symmetric stable process of index α, $1 < \alpha \leq 2$, the characteristic function of each $X(t)-X(s)$ is given by

$$E\,\mathrm{e}^{\mathrm{i}\gamma(X(t)-X(s))} = \mathrm{e}^{-\|X(t)-X(s)\|_\alpha^\alpha |\gamma|^\alpha},$$

Cambanis and Soltani (1984), and therefore (2.1) becomes equivalent to

$$\int_0^1 \frac{\mathrm{d}s}{\|X(t)-X(s)\|_\alpha} < \infty \quad \text{for a.e. } t \in [0, 1], \tag{2.4}$$

and if

$$\int_0^1 \int_0^1 \frac{\mathrm{d}s\,\mathrm{d}t}{\|X(t)-X(s)\|_\alpha} < \infty, \tag{2.5}$$

then $\alpha(1, x)$ is in $L^2(\mathrm{d}x)$, Berman (1969, p. 283).

Conditions (2.2) through (2.5) are in terms of the variances (or α-norms) of the increments. We will use them indirectly in Theorem 3.4 to exhibit that for the moving average processes the local behaviours of the function f have influences on the existence of local time.

3. Local times for stable moving average processes

With the same notations as in Section 2, let us first start with the following lemma.

Lemma 3.1. *Let $X(t, \omega)$, $t \in [0, 1]$, be a stochastic process and δ, $0 < \delta \leq 1$, be a fixed real number. Suppose for any given interval $I \subset [0, 1]$, $\lambda(I) \leq \delta$, with probability one, $X(t, \omega)$ has a local time $\alpha(t, x)$ on I which is continuous in $t \in I$, for a.e. x. Then with probabilty one $X(t, \omega)$ has a local time on $[0, 1]$ which is continuous in t for a.e. x.*

Proof. By the assumption for each $I \subset [0, 1]$ with $\lambda(I) \leq \delta$ there is A_I, $P(A_I) = 1$, such that for each $\omega \in A_I$, and every $t \in I$,

$$\mu_I(t, B) = \int_B \alpha_I(t, x)\,\mathrm{d}x, \quad B \in \mathscr{B}(R).$$

Let $J = \{i_0, \ldots, i_N\}$ be a partition of $[0, 1]$ for which $i_0 = 0$, $i_j - i_{j-1} = \delta$, $j = 1, \ldots, N-1$, $i_N - i_{N-1} \leq \delta$. Then for $\omega \in \bigcap_{j=1}^N A_{I_j}$,

$$\mu(1, B) = \sum_{j=1}^N \mu_{I_j}(i_j, B), \quad B \in \mathscr{B}(R),$$

where $I_j = [i_{j-1}, i_j]$. Therefore $\alpha(1,x) \equiv d\mu(1,x)/dx = \sum_{j=1}^{N} \alpha_{I_j}(i_j, x)$, which is a local time for $X(t,\omega)$ on $[0,1]$. Moreover for each $t_0 \in [0,1]$, there is a j^* with $t_0 \in I_{j^*}$, and therefore

$$\alpha(t_0, x) = \sum_{j=1}^{j^*-1} \alpha_{I_j}(i_j, x) + \alpha_{I_j^*}(t_0, x).$$

If t_0 is an interior point of I_j^*, then the assertion follows by the continuity of $\alpha_{I_j^*}(t,x)$. If t_0 is a boundary point of I_{j^*}, namely $t = i_{j^*-1}$, then for a.e. x,

$$\lim_{t\to t_0^+} \alpha(t,x) = \sum_{j=1}^{j^*-1} \alpha_{I_j}(i_j, x) + \lim_{t\to t_0^+} \alpha_{I_j^*}(t,x) = \sum_{j=1}^{j^*-1} \alpha_{I_j}(i_j, x) = \alpha(t_0, x),$$

because $\alpha_{I_j^*}(t_0, x) = 0$ for a.e. x. Also by the continuity of $\alpha_{I_{j^*-1}}(t,x)$, $\lim_{t\to t_0^-} \alpha(t,x) = \alpha(t_0, x)$ for a.e. x. Now since J is a finite set we obtain that $\alpha(t,x)$ is continuous in t for a.e. $x \in \mathbb{R}$. The proof is complete. □

Now consider the stable moving average process $X(t)$, $t \in \mathbb{R}$, given by (1.1). The following assumptions on the function f will appear in proceeding theorems, thus are stated here separately:

Assumption 3.2. The function f is discontinuous at a point $t_0 \in \mathbb{R}$, $f(t_0^+)$ and $f(t_0^-)$ exist and are finite and distinct.

Assumption 3.3. The function f is continuous at zero and $\lim_{x\downarrow 0} x^{1+\eta-\alpha}|f(x)|^\alpha > 0$ for small $\eta > 0$.

Theorem 3.4. *Suppose $X(t)$, $t \in \mathbb{R}$, is a stable moving average process of index α, $1 < \alpha \leq 2$, given by* (1.1). *If the function f satisfies Assumption* 3.2 *or* 3.3, *then almost every sample function of $X(t)$, $t \in \mathbb{R}$, has a local time $\alpha(t,x)$ on* $[0,1]$ *which is continuous in t, $0 \leq t \leq 1$ for a.e. $x \in \mathbb{R}$.*

Proof. First let f satisfy Assumption 3.2, and $t_0 \in (0,\infty)$, by our assumption $f(t_0^+)$ and $f(t_0^-)$ exist, are finite and $f(t_0^+) \neq f(t_0^-)$. Let $\varepsilon < |f(t_0^+) - f(t_0^-)|$. There are $\delta_1 > 0$, $\delta_2 > 0$ such that $|f(t_0^-) - f(y)| < \frac{1}{2}\varepsilon$ for $y \in (t_0 - \delta_1, t_0)$ and $|f(t_0^+) - f(y)| < \frac{1}{2}\varepsilon$ for $y \in (t_0, t_0 + \delta_2)$. Let $\delta = \min(\delta_1, \delta_2)$, then for each y_1 and y_2 with $t_0 - \delta < y_1 < t_0$, $t_0 < y_2 < t_0 + \delta$,

$$|(f(y_2) - f(y_1)) - (f(t_0^+) - f(t_0^-))| < \varepsilon.$$

This gives that

$$|f(t_0^+) - f(t_0^-)| - \varepsilon < |f(y_2) - f(y_1)| < \varepsilon + |f(t_0^+) - f(t_0^-)|, \tag{3.1}$$

for $t_0 - \delta < y_1 < t_0 < y_2 < t_0 + \delta$.

Now for the moving average process given by (1.1),

$$\|X(t)-X(s)\|_\alpha^\alpha=\int_0^\infty |f(y+|s-t|)-f(y)|^\alpha\,\mathrm{d}y+\int_0^{|s-t|}|f(y)|^\alpha\,\mathrm{d}y$$
$$\geq \int_0^\infty |f(y+|s-t|)-f(y)|^\alpha\,\mathrm{d}y$$
$$\geq \int_{t_0-|s-t|}^{t_0} |f(y+|s-t|)-f(y)|^\alpha\,\mathrm{d}y.$$

But note that if $y\in(t_0-|s-t|, t_0)$, then $y+|s-t|\in(t_0, t_0+|s-t|)$. Also for $|s-t|<\delta$, we have $(t_0-|s-t|, t_0)\subseteq(t_0-\delta, t_0)$, $(t_0, t_0+|s-t|)\subseteq(t_0, t_0+\delta)$. Therefore, by applying (3.1) it follows that

$$\int_{t_0-|s-t|}^{t_0}|f(y+|s-t|)-f(y)|^\alpha\,\mathrm{d}y>[|f(t_0^+)-f(t_0^-)|-\varepsilon]^\alpha|s-t|$$

as long as $|s-t|<\delta$. This gives that for $|s-t|<\delta$,

$$\|X(t)-X(s)\|_\alpha^\alpha>[|f(t_0^+)-f(t_0^-)|-\varepsilon]^\alpha|s-t|. \tag{3.2}$$

Now let $I=[a,b]$ be an interval in $[0,1]$ with $b-a<\delta$. Then for every $t\in I$,

$$\int_a^b\frac{\mathrm{d}s}{\|X(t)-X(s)\|_\alpha}$$
$$\leq[|f(t_0^+)-f(t_0^-)|-\varepsilon]^{-1}\int_a^b|s-t|^{-1/\alpha}\,\mathrm{d}s$$
$$=\frac{\alpha}{\alpha-1}[|f(t_0^+)-f(t_0^-)|-\varepsilon]^{-1}[(t-a)^{1-1/\alpha}+(b-t)^{1-1/\alpha}]$$

which is finite for every $t\in I$. Therefore (2.4) is satisfied with $[0,1]$ replaced by I and the conclusion for $t_0\in(0,\infty)$ follows by applying Geman's theorem (Geman, 1976, Theorem B), on I and using Lemma 3.1.

Next assume $t_0=0$. then by our assumption, $|f(0^+)|>0$, and for $\varepsilon<|f(0^+)|$ there is $\delta>0$ such that $|f(0^+)|-\varepsilon<|f(y)|$, whenever $0<y<\delta$. Therefore for each s, $t\in[0,1]$ with $|s-t|<\delta$ we have

$$\|X(t)-X(s)\|_\alpha^\alpha\geq\int_0^{|s-t|}|f(y)|^\alpha\,\mathrm{d}y\geq|s-t|[|f(0^+)|-\varepsilon]^\alpha. \tag{3.3}$$

This gives that for any interval $[a,b]\subseteq[0,1]$ with $b-a<\delta$, and $t\in[a,b]$,

$$\int_a^b\frac{\mathrm{d}s}{\|X(t)-X(s)\|_\alpha}\leq[|f(0^+)|-\varepsilon]^{-1}\int_a^b|s-t|^{-1/\alpha}\,\mathrm{d}s$$
$$=\frac{\alpha}{\alpha-1}[|f(0^+)-\varepsilon]^{-1}[(t-a)^{1-1/\alpha}+(b-t)^{1-1/\alpha}]$$

which is finite for every $t\in[a,b]$. Again Geman's theorem and Lemma 3.1 give the desired result.

Next let Assumption 3.3 be satisfied, i.e. $\lim_{x\to 0^+} x^{1+\eta-\alpha}|f(x)|^\alpha > 0$. This implies that for any $x \in (0, \delta)$, $|f(x)|^\alpha > Kx^{\alpha-1-\eta}$ for some $\delta > 0$ and constant $K > 0$. This gives that for $|s-t| < \delta$,

$$\|X(s)-X(t)\|_\alpha^\alpha \geq \int_0^{|s-t|} |f(y)|^\alpha \,\mathrm{d}y \geq (\alpha-\eta)^{-1}K|s-t|^{\alpha-\eta}. \tag{3.4}$$

Therefore for any interval $[a, b] \subseteq [0, 1]$ with $b - a < \delta$, and every $t \in [a, b]$,

$$\begin{aligned}\int_a^b \frac{\mathrm{d}s}{\|X(t)-X(s)\|_\alpha} &\leq \int_a^b \left[\int_0^{|s-t|} |f(y)|^\alpha \,\mathrm{d}y\right]^{-1/\alpha} \mathrm{d}s \\ &\leq (\alpha-\eta)^{1/\alpha} K^{-1/\alpha} \int_a^b |s-t|^{-(\alpha-\eta)/\alpha} \,\mathrm{d}s \\ &= (\alpha/\eta)(\alpha-\eta)^{1-\alpha} K^{-1/\alpha}[(t-a)^{\eta/\alpha} + (b-t)^{\eta/\alpha}]\end{aligned}$$

which is finite for every $t \in [a, b]$. Again we are in a position to apply both Geman's theorem and Lemma 3.1 to conclude the result. The proof of the theorem is complete. □

Corollary 3.5. *Let $X(t)$, $t \in \mathbb{R}$, be a stable moving average processes of index α, $1 < \alpha \leq 2$, given by* (1.1). *If the function f satisfies Assumption* 3.2 *or* 3.3, *then with probability one every $X(t, \omega)$, $t \in \mathbb{R}$, is a Jarnik function (J_1) and the set $\{s \in \mathbb{R};\ X(t, \omega) = X(s, \omega)\}$ is uncountable for a.e. $t \in \mathbb{R}$.* □

Amplification. As it was suggested by a referee and follows from the proof of Theorem 3.4, this theorem remains valid if Assumptions 3.2 and 3.3 are replaced by the following weaker assumption. Namely there is a point $t_0 \in \mathbb{R}$ and $0 < \eta < \alpha$ such that

$$\liminf_{\substack{|t_2-t_1|\to 0\\ t_1<t_0<t_2}} |t_2-t_1|^{1+\eta-\alpha}|f(t_2)-f(t_1)|^\alpha > 0. \tag{3.5}$$

4. Hölder conditions and γ-variation

Let $F(t)$, $0 \leq t \leq 1$, be as in Section 2, following Berman (1969b), the γ-variation ($\gamma \geq 1$) of the function $F(t)$ on $I = [a, b]$, $0 \leq a < b \leq 1$, is defined as

$$\lim_{n\to\infty} \sup_{a\leq t_1\leq\cdots\leq t_n\leq b} \sum_{j=1}^{n-1} |F(t_{j+1}) - F(t_j)|^\gamma.$$

the function $F(t)$, $0 \leq t \leq 1$, is said to satisfy a Hölder condition of order q at t_0 in $[0, 1]$ if there are positive constants c and ε such that $|F(t) - F(t_0)| \leq c|t - t_0|^q$ whenever $|t - t_0| \leq \varepsilon$.

The following theorem in the Gaussian case is due to Berman (1969b, Theorem 5.1). It is straighforward to modify Berman's proof to the stable case. Indeed Berman first developed a theory on local times of functions and applied it to a Gaussian process $X(t, \omega)$ by means of the following formula:

$$E\int_{-\infty}^{+\infty}|u|^p|g(u)|^2\,\mathrm{d}u$$
$$= C(p)\int_I\int_I[E(X(s)-X(t))^2]^{-(p+1)/2}\,\mathrm{d}s\,\mathrm{d}t,\quad p\geq 0, \tag{4.1}$$

where $g(u,\omega)=\int_I \mathrm{e}^{\mathrm{i}uX(t,\omega)}\,\mathrm{d}t$, $-\infty<u<\infty$, and $C(p)$ is a constant depending only on p. When $X(t,\omega)$, $t\in I$, is symmetric stable process of index α, $1<\alpha\leq 2$, (4.1) becomes

$$E\int_{-\infty}^{+\infty}|u|^p|g(u)|^2\,\mathrm{d}u$$
$$= K(p)\int_I\int_I\|X(t)-X(s)\|_\alpha^{-(p+1)}\,\mathrm{d}s\,\mathrm{d}t,\quad p\geq 0. \tag{4.2}$$

Theorem 4.1. *Let $X(t,\omega)$ be a symmetric stable process of index α, $1<\alpha\leq 2$, on $[0,1]$. If the integral on the right-hand side of (4.2) is finite for an interval $I\subset[0,1]$ then for almost all ω;*

(a) *The γ-variation of $X(\cdot,\omega)$ on the interval $I\subset[0,1]$ is infinite for $\gamma=p+1$.*

(b) *$X(\cdot,\omega)$ nowhere satisfies a Hölder condition of order $2/(p+1)$ on I.* □

The following theorem is concerned with the Hölder condition and γ-variation of sample paths of moving average processes.

Theorem 4.2. *Let $X(t)$, $t\in\mathbb{R}$, be the symmetric stable moving average process, of index α, $1<\alpha\leq 2$, given by (2). Then:*

(A) *If Assumption 3.2 is satisfied then for almost all ω:*

(a) *The γ-variation of $X(\cdot,\omega)$ on any interval $I\subseteq[0,1]$ is infinite for any $\gamma\in[1,\alpha)$.*

(b) *$X(\cdot,\omega)$ nowhere satisfies a Hölder condition of order $q>2/\alpha$.*

(B) *If Assumption 3.3 is satisfied, then for almost all ω:*

(a) *The γ-variation of $X(\cdot,\omega)$ on any interval $I\subseteq[0,1]$ is infinite for any $\gamma\in[1,\alpha/(\alpha-\eta))$.*

(b) *$X(\cdot,\omega)$ nowhere satisfies a Hölder condition order $q>2(1-\eta/\alpha)$.*

Proof. (A) Under Assumption 3.2 it follows from (3.2) and (3.3) that there is $\delta>0$ for which for any interval $I\subseteq[0,1]$ with $\lambda(I)\leq\delta$,

$$\int_I\int_I\|X(t)-X(s)\|_\alpha^{-(p+1)}\,\mathrm{d}s\,\mathrm{d}t<\infty, \tag{4.3}$$

as long as $p+1<\alpha$. Now (A)(a), (A)(b) for such intervals are followed by Theorem 4.1. But $[0,1]$ can be covered by finite number of intervals of length less than δ, which gives the desired result.

(B) Under Assumption 3.3, it follows from (3.4) that (4.3) is satisfied as long as $p+1<\alpha/(\alpha-\eta)$. This with a similar argument as in part (A) gives the result. The proof of the theorem is complete. □

Part (A)(b) (resp. (B)(b)) of Theorem 4.2 can be amplified by using Theorem 2.1 in Geman and Horowitz (1980) which gives that, under the same assumptions, for every t_0 and every $c>0$, the density of the set $\{t: |X(t)-X(t_0)|\leqslant c|t-t_0|^q\}$ is zero for $q>2/\alpha$ (resp. $q>2(1-\eta/\alpha)$). This implies even no approximate Hölder condition at t_0. Also note that part (B) also follows from assumption (3.5) which is a weaker assumption.

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